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ON PROPAGATION OF CONCENTRATION DISTURBANCES IN A MAGNETICALLY STABILIZED FLUIDIZED BED

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Abstract—Propagation of one-dimensional disturbances of solid concentration in liquid–solid and gas–solid fluidized beds of magnetic particles in an external magnetic field is considered. Linear disturbances are analysed and the criteria of magnetic stabilization of liquid–solid and gas–solid fluidized beds are derived. Propagation of non-linear long concentration waves is analysed. Burgers equation is shown to describe the propagation of long waves in a fluidized bed of magnetic particles. Formation of domains with a sharp change in solid concentration ("shock" fronts) is analysed. The structure of the shock front (in particular the thickness of the front) is found to depend on the magnetic parameters of the solid particles. The obtained results can be used to explain the mechanism of supression of bubble formation in magnetically stabilized fluidized beds.

Key Words: fluidized beds, magnetic stabilization, concentration waves, bubble formation

INTRODUCTION

The first fundamental theoretical study of the problem of magnetic stabilization of a uniform fluidized bed of magnetizable particles has been given by Rosensweig (1979) (see also the monograph by Rosensweig 1985). Propagation of linear waves in a gas–solid fluidized bed of magnetic particles in an external magnetic field has been considered and the criteria of magnetic stabilization have been obtained. Later the proposed model was generalized by Rosensweig & Cyprios (1991) for liquid–solid fluidized beds and the criteria of linear stability for systems of magnetizable particles in a neutral fluid as well as for systems of neutral particles in a magnetic fluid have been derived.

Since the effect of stabilization is due to the interaction between magnetizable particles, some ideas proposed by Foscolo *et al.* (1985), for the description of wave phenomena in a fluidized bed of interacting particles, can be used to understand the mechanism of magnetic stabilization.

The linear theory by Rosensweig requires further development in order to describe the detailed structure of the propagating linear concentration disturbances.

Even more interesting a problem is to develop Rosensweig's model in order to analyse the propagation of non-linear disturbances of solid concentration in a fluidized bed of magnetic particles. The basic purpose in the analysis of non-linear waves is to describe the formation of concentration discontinuities (concentration "shock" waves) and fronts with a sharp change of solid concentration. The mechanism of formation of such domains can be considered as a qualitative model of bubble formation, while structures of concentration wave fronts were found to give certain ideas on the structures of bubble boundaries in fluidized beds. This means that changes in the structures of wave fronts induced by a magnetic field can give useful information on the mechanism of magnetic stabilization of bubbling fluidized beds.

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BASIC EQUATIONS

We consider the propagation of one-dimensional disturbances of solid concentration in the vertical direction in a fluidized bed of magnetic particles in an external magnetic field. To describe the motion of the fluid (gas) and particles, the model of dispersion, as a double continuum consisting of two mutually penetrating and interacting ideal fluids, is used.

Solid particles are assumed to be spherical and to have equal diameter d_p. The particle size is assumed to be small so that the Reynolds number $\mathrm{Re} = Ud_p/\nu \ll 1$, where U is the superficial fluid (gas) velocity in the undisturbed (uniform) fluidized bed and ν is the kinematic viscosity of the fluid (gas). In accordance with the above assumptions an interphase interaction is supposed to be linear with respect to the relative velocity of the liquid (gas) and solid phase.

The mass conservation equations for the liquid (gas) and solid phase are as follows:

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial(\epsilon v_f)}{\partial z} = 0 \qquad [1]$$

$$\frac{\partial \alpha}{\partial t} + \frac{\partial(\alpha v_p)}{\partial z} = 0 \qquad [2]$$

$$\epsilon + \alpha = 1 \qquad [3]$$

where ϵ is the void fraction, α is the volumetric concentration of solid phase, v_f and v_p are the interstitial fluid (gas) velocity and the mean velocity of solid particles and z is the vertical coordinate.

The momentum conservation equations for the liquid (gas) and solid phase can be written as follows:

$$\rho_f \epsilon \left(\frac{\partial v_f}{\partial t} + v_f \frac{\partial v_f}{\partial z} \right) = -\frac{\partial p_f}{\partial z} - \rho \epsilon_f g + F_I \qquad [4]$$

$$\rho_p \alpha \left(\frac{\partial v_p}{\partial t} + v \frac{\partial v_p}{\partial z} \right) = -\frac{\partial p_p}{\partial z} - \rho_p \alpha g - F_I + f_m \qquad [5]$$

where ρ_f and ρ_p are the fluid and solid densities, respectively, p_f is the fluid pressure, p_p is the effective pressure of the solid phase, F_I is the interphase interaction force, f_m is the magnetic force and p_p is the effective pressure in the pseudogas of the solid particles.

Closure of these equations can be achieved by constitutive assumptions for the interaction force F_I, magnetic force f_m and effective pressure p_p.

Assuming that the solid particles are small so that $\mathrm{Re} \ll 1$, the interphase interaction force can be written in the usual form

$$F_I = \alpha \frac{\partial p}{\partial z} + 18 \frac{\rho_p \nu}{d_p^2} \alpha \Phi(\epsilon) u \qquad [6]$$

where

$$u = v_f - v_p$$

is the relative velocity of fluid and particles. Following Rosensweig (1979, 1985), we also assume that the interphase interaction force does not depend on the vector of magnetization of solid particles. In particular this assumption is approved for spherical particles (Rosensweig 1979, 1985). With the above assumptions the function $\Phi(\epsilon)$ can be written in the Richardson–Zaki (1954) form

$$\Phi(\epsilon) = \epsilon^{-n} \qquad [7]$$

where $n = 2.8$.

Now we give the constitutive relationship for the magnetic force. Following Rosensweig (1979, 1985) we assume that: (a) effects of magnetostriction can be neglected; (b) the solid material is magnetically soft so that hysteresis phenomena can be neglected as well. In accordance with

Rosensweig (1979, 1985) and Cowley & Rosensweig (1967), the magnetic force can be represented in the Kelvin form

$$f_m = \mu_0 \alpha M_p(H_p) \frac{\partial H_p}{\partial z} \qquad [8]$$

where μ_0 is the magnetic permeability of the vacuum, H_p is the average local value of the magnetic field strength in the solid phase and $M_p(H_p)$ is the magnetization of the solid material.

The general form of $M_p(H_p)$ with the above assumptions is given in figure 1 where M_s and H_s are the magnetization and the field strength of the magnetic saturation of the solid material.

It is also supposed here that the magnetic field strength is not very high so that we can neglect the formation of vertical "strings" of solid particles. The deep analysis of this phenomenon has been recently given by Zimmel *et al.* (1991), although some earlier publications dedicated to this subject can be pointed out as well.

The above assumptions actually mean that constitutive relationships can be written in the form [6]–[8] so that the consideration given below is framed by Rosensweig's model of the magnetically stabilized fluidized beds.

The hydrodynamic equations [1]–[5] and the constitutive relationships [6]–[8] should be considered together with the equations of magnetic field. While the magnetic field applied to the bed of particles is uniform, the bed magnetization and the magnetic field are related to the corresponding phase parameters as follows (Rosensweig 1991):

$$H + M = B_0/\mu_0, \quad M = \epsilon M_f + \alpha M_p, \quad H = \epsilon H_f + \alpha H_p \qquad [9a]$$

$$M_f = \chi_f(H_f)H_f, \quad M_p = \chi_p(H_p)H_p, \quad M = \chi H \qquad [9b]$$

where H_f is the average local strength of the magnetic field in the fluid phase, $M_f(H_f)$ is the magnetization of fluid, χ_p and χ_f are the chord magnetic susceptibilities of the phases; the parameters without a subscript denote the corresponding values for the mixture. The relationship between susceptibilities is given by the generalization of the Clausius–Mosotti formula following from the mean-field theory (see Landauer 1978) in the form

$$\frac{\chi - \chi_f}{\chi + 2\chi_f + 3} = \alpha \frac{\chi_p - \chi_f}{\chi_p + 2\chi_f + 3} \qquad [10]$$

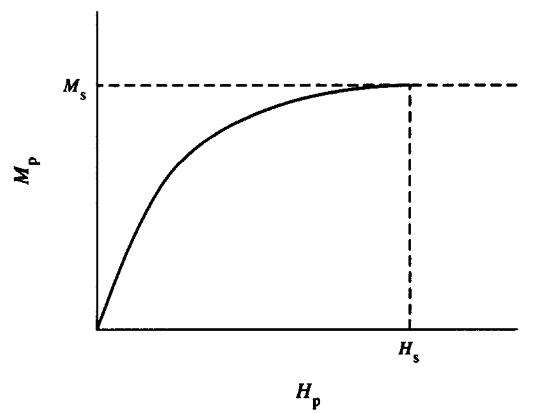

Figure 1. The general form of magnetization as a function of the magnetic field strength (the subscript "s" corresponds to the magnetic saturation).

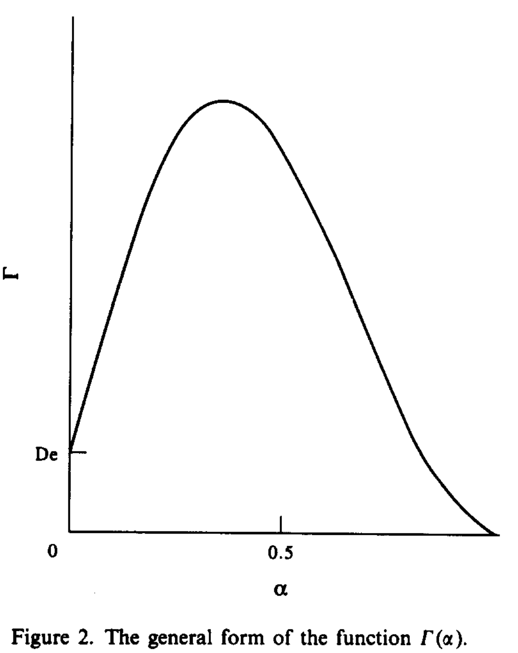

Figure 2. The general form of the function $\Gamma(\alpha)$.

Below we consider the case when only particles are magnetizable, not fluid, so that $\chi_f = M_f = 0$. In this case, from [9] and [10] follows the equation connecting the magnetic parameters of the solid phase H_p and M_p with the solid concentration α in the form

$$H_p + \frac{1+2\alpha}{3} M_p = \frac{B_0}{\mu_0} \quad [11]$$

When the solid material is magnetically saturated ($H_p > H_s$) so that $M = M_s$, the last equation gives the following relationship between the gradients of solid concentration and magnetic field:

$$\frac{\partial H_p}{\partial z} = -\tfrac{2}{3} M_s \frac{\partial \alpha}{\partial z} \quad [12]$$

so that the magnetic force becomes

$$f_m = -\tfrac{2}{3}\mu_0 M_s^2 \alpha \frac{\partial \alpha}{\partial z} \quad [13]$$

To close the system [1]–[5], it is necessary to add constitutive relationships and/or differential equations for the effective pressure of the solid phase p_p. Some simple constitutive relationships and estimations for p_p will be given in the following sections of this paper.

We now introduce the dimensionless variables

$$z^* = \frac{z}{L}, \quad t^* = \frac{U}{L} t, \quad v_f^* = \frac{v_f}{U}, \quad v_p^* = \frac{v_p}{U} \left(u^* = \frac{u}{U} \right) \quad [14a]$$

$$p_f^* = \frac{p_f}{\rho_f U^2}, \quad p_p^* = \frac{p_p}{\rho_p U^2}, \quad H_p^* = \frac{\mu_0 H_p}{B_0}, \quad M_p^* = \frac{\mu_0 M_p}{B_0} \quad [14b]$$

where L is the linear scale of disturbance.

Taking into account the constitutive relationships, the dimensionless mass, momentum conservation and magnetic field equations can be reduced to (the superscript * is henceforth omitted)

$$\epsilon_t + (\epsilon v_f)_z = 0 \quad [15]$$

$$\alpha_t + (\alpha v_p)_z = 0 \quad [16]$$

$$(v_f)_t + v_f(v_f)_z = -(p_f)_z + \mathrm{Fr}[-1 - \mathrm{De}^{-1}\kappa\alpha\epsilon^{-1}\Phi(\epsilon)u] \quad [17]$$

$$(v_p)_t + v_p(v_p)_z = -\mathrm{De}(p_f)_z - \alpha^{-1}(p_p)_z + \mathrm{Fr}[-1 + \kappa\Phi(\epsilon)u - mM_p(H_p)(H_p)_z] \quad [18]$$

The magnetic field strength as a function of α can be found from [9]–[10] (or [11] for the saturated bed). In [15]–[18] the density (De) and Froude (Fr) numbers, and other dimensionless groups, are given by

$$\mathrm{De} = \frac{\rho_f}{\rho_p}, \quad \mathrm{Fr} = \frac{gL}{U^2}, \quad \kappa = \frac{18\rho_f \nu U}{\rho_p g d_p^2} = 18 \frac{\rho_f}{\rho_p} \frac{L}{d_p} \frac{1}{\mathrm{Re}} \frac{1}{\mathrm{Fr}}, \quad m = \frac{B_0^2}{\mu_0 \rho_p g L} \quad [19]$$

Following Kurdyumov & Sergeev (1987) and Sergeev (1988), from the equations of mass conservation with the assumption that the two-phase flow is undisturbed as $z \to \infty$ we find the following relationship between the relative velocity, the velocity of solid phase and the voidage:

$$u = \frac{1 - v_p}{\epsilon} \quad [20]$$

To determine the parameters of the steady state we assume

$$\alpha = \alpha_0 = \text{const}, \quad \epsilon = \epsilon_0 = 1 - \alpha_0, \quad v_f = \epsilon_0^{-1}, \quad v_p = 0$$

$$H_p = H_{p0} = \text{const}, \quad p_p = p_{p0} = \text{const} \quad [21]$$

The equation for the uniform steady voidage ϵ_0 is found from [17] and [18] to be

$$1 - \mathrm{De} = \kappa\phi(\epsilon_0) \quad (\mathrm{De} < 1) \quad [22]$$

where

$$\phi(\epsilon) = \epsilon^{-2}\Phi(\epsilon) = \epsilon^{-n-2} \quad [23]$$

Obviously the magnetic field does not affect the uniform steady void fraction.

PROPAGATION OF LINEAR CONCENTRATION DISTURBANCES

In this section, following Rosensweig (1979, 1985) we neglect the particle pressure term p_p. It should be noted that the definitive criteria for the magnetic stabilization of fluidized beds can be obtained using the simplest constitutive relationship $p_p = 0$, while the detailed analysis of the effective solid pressure in magnetic systems requires considerable development of the theory of microscale motion in the solid phase.

We linearize [15]–[18] in the vicinity of the uniform steady state [21] introducing small disturbances

$$\alpha = \alpha_0 + \eta, \quad v_f = \epsilon_0^{-1} + v_f^1, \quad v_p = v_p^1, \quad H = H_{p0} + H^1 \quad [24]$$

so that

$$\eta, v_f^1, v_p^1, H^1 \ll 1 \quad [25]$$

It should be noted that the linearized dimensionless magnetic force $f_m = -m\mathrm{Fr}M_p(H_p)(H_p)_z$ in [18] reduces to

$$f_m = -\gamma\eta_z; \quad \gamma = \frac{2}{3}\frac{\mu_0 M_{p0}^2}{\rho_p U^2[1 + \frac{1}{3}(1 + 2\alpha_0)\hat{\chi}_{p0}]} \quad [26]$$

where $M_{p0} = M_p(H_{p0})$. Here we introduce the tangent susceptibility of the solid material

$$\hat{\chi}_p = \frac{dM_p}{dH_p} = \chi_p + H_p\frac{d\chi_p}{dH_p}; \quad \hat{\chi}_{p0} = \hat{\chi}_p(H_{p0}) \quad [27]$$

The linearized equations for the system [15]–[18] can be reduced to the following equations for the disturbances of solid concentration (voidage) and the velocities of liquid (gas) and solid phases:

$$\eta_t + \epsilon_0^{-1}\eta_z - \epsilon_0(v_f^1)_z = 0, \quad \eta_t + \alpha_0(v_p^1)_z = 0 \quad [28]$$

$$(v_p^1)_t - \mathrm{De}((v_f^1)_t + \epsilon_0^{-1}(v_f^1)_z) + \kappa\mathrm{Fr}(\phi_0 v_p^1 + \phi_0'\eta) + \gamma\eta_z = 0 \quad [29]$$

where

$$\phi_0 = \phi(\epsilon_0), \quad \phi_0' = \left(\frac{d\phi}{d\epsilon}\right)_{\epsilon=\epsilon_0} \quad [30]$$

From [28] and [29] it follows the equation for the disturbance of solid concentration (voidage) η in the form

$$\xi\left(\frac{\partial}{\partial t} + c_1\frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial t} + c_2\frac{\partial}{\partial z}\right)\eta + \left(\frac{\partial}{\partial t} + c_0\frac{\partial}{\partial z}\right)\eta = 0 \quad [31]$$

where

$$\xi = \frac{\epsilon_0^{n+2}(\epsilon_0 + \mathrm{De}\alpha_0)}{\kappa\mathrm{Fr}}, \quad c_0 = \frac{(n+2)\alpha_0}{\epsilon_0} \quad [32]$$

We are reminded here that for the Richardson–Zaki correlation, $n = 2.8$. It will be noted in the next section of this article that c_0 is the velocity of kinematic (long) concentration wave at the parameters of the uniform steady state. The characteristic velocities of the higher order c_1 and c_2 are of the form

$$c_{1,2} = \frac{\alpha_0\epsilon_0}{\epsilon_0 + \mathrm{De}\alpha_0}\left(-\frac{\mathrm{De}}{\epsilon_0^2} \pm \frac{1}{\alpha_0^{1/2}}\left(\gamma + \frac{\mathrm{De}}{\epsilon_0^3}(\alpha_0\epsilon_0^2\gamma - 1)\right)^{1/2}\right) \quad [33]$$

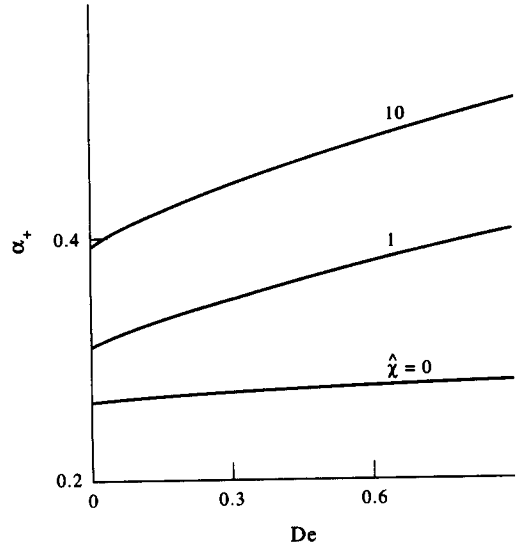

Figure 3. The concentration α_+ corresponding to the maximum of $\Gamma(\alpha)$ as a function of density ratio De.

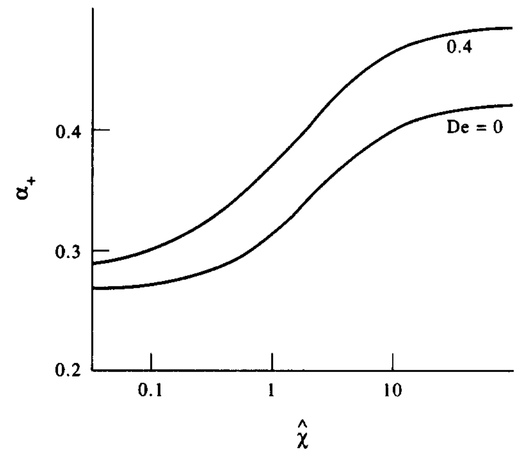

Figure 4. α_+ as a function of magnetic susceptibility.

Since for any values of parameters

$$c_1 > 0, \quad c_2 < 0 \qquad [34]$$

the steady state of a fluidized bed is linearly stable while $c_1 > c_0$ (see, for example, Witham 1974). Incorporating [34] and [35] the stability criterion becomes

$$\gamma > \frac{\alpha_0((n+2)(\epsilon_0 + \mathrm{De}\alpha_0) + \mathrm{De})^2 + \mathrm{De}\epsilon_0}{\epsilon_0^3(\epsilon_0 + \mathrm{De}\alpha_0)} \qquad [35]$$

In order to analyse the stability criterion [35] in dependence on the physical parameters of phases and the magnetic field we note that the gas velocity U in the expression for the dimensionless parameter γ cannot be given independently on the steady voidage ϵ_0. The correlation between the superficial gas velocity U, the void fraction ϵ_0 and the parameters of the phases in the case of a small Reynolds number based on the solid particle size follows from the steady state momentum equations in the form

$$U = \frac{g d_p^2(\rho_p - \rho_f)\epsilon_0^{n+2}}{18 d_f \nu} \qquad [36]$$

The resulting stability criterion becomes:

$$N > \Gamma(\alpha_0, \mathrm{De}, \hat{\chi}_0) \qquad [37]$$

where

$$\Gamma(\alpha, \mathrm{De}, \hat{\chi}) = \epsilon^{n-1}(1 + \hat{\chi}\alpha)\{-(n+2)^2(1-\mathrm{De})\alpha^2 + (n+2)[n + 2(1+\mathrm{De})]\alpha + \mathrm{De}\} \qquad [38]$$

The dimensionless parameter introduced in [37]

$$N = \frac{18\sqrt{\mu_0}\rho_f \nu M_p(H_{p0})}{g d_p^2 \sqrt{d_p}(\rho_p - \rho_f)} \qquad [39]$$

depends only on the magnetic properties, the size of solid particles and the densities of both phases. The function $\Gamma(\alpha)$ has the form represented in figure 2. At $\alpha = 0$ $\Gamma = \mathrm{De}$. The function $\Gamma(\alpha)$ has a maximum Γ_m at $\alpha = \alpha_+(\chi, \mathrm{De})$ repersented in figures 3 and 4 as a function of De and χ, respectively (it should be noted that the values of De close to 1.0 do not have too much sense in fluidized beds). For gas–solid fluidized beds (De = 0) from the equation $\partial\Gamma/\partial\alpha = 0$ it follows:

$$\alpha_+ = \frac{1}{2\hat{\chi}(n+2)}\{\sqrt{(n+1)^2 + 4\hat{\chi}(1+\hat{\chi})} + 2\hat{\chi} - n - 1\} \qquad [40]$$

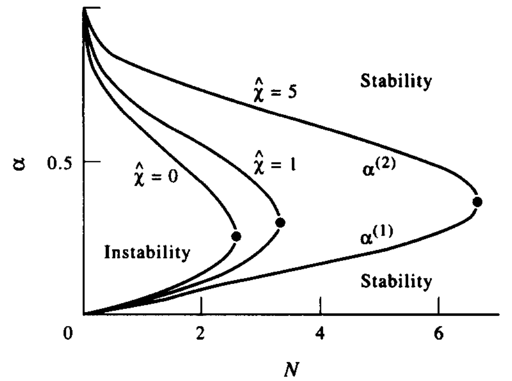

Figure 5. Neutral curves at De = 0. Lower branches correspond to $\alpha^{(1)}$, upper branches to $\alpha^{(2)}$.

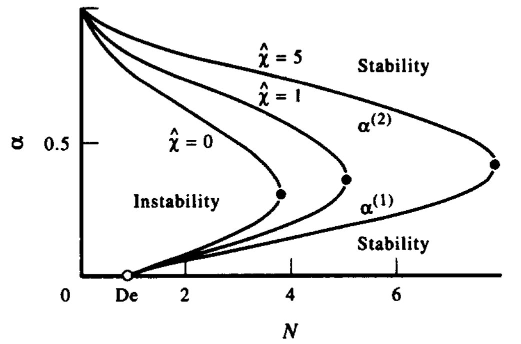

Figure 6. Neutral curves at De = 0.4.

For magnetically saturated gas–solid beds ($\hat{\chi} \to 0$) yields

$$\alpha_+ = \frac{1}{n+1} \quad [41]$$

so that $\alpha_+ \simeq 0.267$ while the Richardson–Zaki drag law is used ($n = 2.8$).

For magnetically saturated liquid–solid fluidized beds the value of α_+ can be found from the equation

$$\alpha_+^2 - \frac{n+2}{(n+1)(1-\text{De})}\alpha_+ + \frac{(n+2)^2 + (n+5)\text{De}}{(n+2)^2(n+1)(1-\text{De})} = 0 \quad [42]$$

When the function $\Gamma(\alpha)$ is incorporated, the stability criterion [37] gives the neutral curves represented in the (α, N)-plane in figures 5 and 6 for De = 0 and De = 0.4, respectively.

The following results can be immediately deduced from the form of the neutral curves.

(1) The uniform state of a fluidized bed can be magnetically stabilized in the whole range of solid concentration (and, respectively, the fluid velocity) by a relatively strong magnetic field such that $N > N_+(\hat{\chi}, \text{De})$, where

$$N_+ = \max_\alpha \Gamma(\alpha).$$

The value of N_+ as a function of De and $\hat{\chi}$ is represented in figures 7 and 8, respectively.

(2) While $\text{De} < N < N_+$, stabilization occurs in the following two intervals of solid concentration:

$$0 \leqslant \alpha < \alpha^{(1)}(N, \hat{\chi}, \text{De}) \quad \text{and} \quad \alpha > \alpha^{(2)}(N, \hat{\chi}, \text{De}) \quad [43]$$

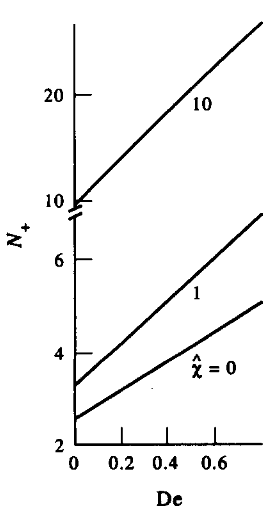

Figure 7. $N_+ = \max_\alpha \Gamma(\alpha)$ as a function of De.

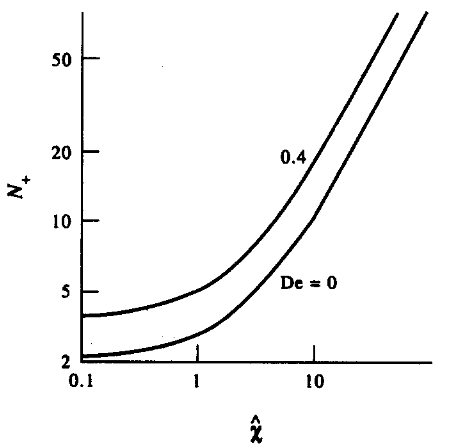

Figure 8. N_+ as a function of $\hat{\chi}$.

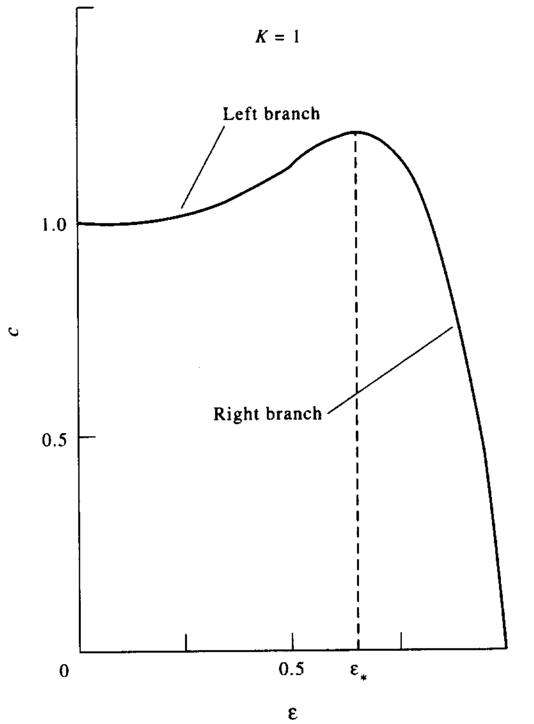

Figure 9. Kinematic wave velocity as a function of void fraction ($K = 1$).

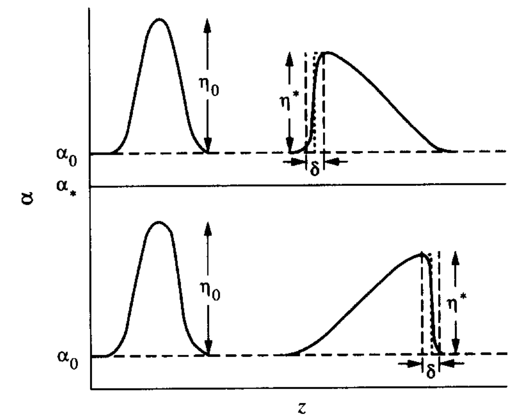

Figure 10. Schematic representation of the propagation of a long concentration wave; top: $\alpha > \alpha_*$ ($\epsilon < \epsilon_*$); bottom: $\alpha < \alpha_*$ ($\epsilon > \epsilon_*$). η_0, initial amplitude; η^*, amplitude of the concentration disturbance; δ, thickness of the wave front. ...., Shock formation of neutral particles in a fluidized bed.

The critical concentrations $\alpha^{(1)}$ and $\alpha^{(2)}$ are given, respectively, by the lower and upper branches of the neutral curves (see figures 5 and 6; the black points correspond to $\alpha = \alpha^{(1)} = \alpha^{(2)} = \alpha_+, N = N_+$). The first interval in [43] corresponds to the relatively diluted two-phase dispersion, while the second corresponds to the dense fluidized bed. However, we must underline, that the maximum solid concentration in a fluidized bed is $\alpha = \alpha_{\mathrm{mf}} \simeq 0.63$ so that the second interval should be replaced by $\alpha_{\mathrm{mf}} > \alpha > \alpha^{(2)}$. This gives the following additional condition for the magnetic stabilization of a dense fluidized bed: $N > N_*(\mathrm{De}, \hat{\chi})$, where $N_* = \Gamma(\alpha_{\mathrm{mf}}, \mathrm{De}, \hat{\chi})$. Incorporating $n = 2.8$ and $\alpha_{\mathrm{mf}} \simeq 0.63$, the last condition can be written as

$$N > N_* \simeq 0.92(1 + 0.63\hat{\chi})(1 + 3.12\mathrm{De}) \qquad [44]$$

The concentration $\alpha^{(2)}$ can be related to the minimum bubbling point. Incorporating [36] we now obtain, from [43], two ranges of fluid velocity: $U_{\mathrm{mf}} < U < U_{\mathrm{mb}}$, where $U_{\mathrm{mb}} = U(\epsilon^{(2)})$, $\epsilon^{(2)} = 1 - \alpha^{(2)}$, and $U > U^{(1)}$, both providing magnetic stabilization of the uniform state of a fluidized bed.

(3) In the case of a relatively weak magnetic field such that $0 \leqslant N < \mathrm{De}$, stabilization of a diluted liquid–solid suspension is impossible. Since $N_* > \mathrm{De}$, we can easily conclude that a magnetic stabilization of a dense fluidized bed cannot be achieved in this range of N either.

While $\mathrm{De} \leqslant N < N_+, N > N_*$, for $\alpha^{(1)} < \alpha < \alpha^{(2)}$ propagation of disturbances in the form of weak concentration shocks is possible (Witham 1974). The obtained results mean that the magnetic field obstructs formation of the concentration discontinuities, hence the magnetic field increases the ranges of parameters corresponding to the uniform fluidization.

We now analyse the propagation of the linear concentration wave (such analysis can be based on the problem of a signal propagation as well as on the Cauchy problem, see Witham 1974). From [31], the inequalities [34] and the appropriate boundary and initial conditions it follows (Witham 1974) that in the very first moments after the formation of disturbance (or not far from the source of a disturbance), the wave front propagating with speed c_1 appears. The dimensionless scale of damping of this front is $c_1/(\kappa \mathrm{Fr})$ where κ and Fr are given by [19]. Hence, the characteristic damping time $(\kappa \mathrm{Fr})^{-1}$ does not depend on the magnetic field strength or the magnetic properties of the solid phase. In the initial period the linear scale of damping of disturbances is decreasing with the Reynolds number based on the particle size.

Now we consider the case of high magnetization of solid particles such that

$$\mu_0 M_{p0}^2 \gg \rho_p U^2 \tag{45}$$

In this case the characteristic velocities $c_{1,2}$ can be approximately written as

$$c_{1,2} = \pm \frac{\epsilon_0}{\epsilon_0 + \mathrm{De}\alpha_0} \sqrt{\alpha_0 \gamma} \tag{46}$$

The dimensional expression for the initial speed of the wave front is as follows:

$$c_1 \simeq \frac{\epsilon_0 \sqrt{2\mu_0 \rho_p \alpha_0} M_{p0}}{(\rho_p \epsilon_0 + \rho_f \alpha_0)\sqrt{3 + (1 + 2\alpha_0)\hat{\chi}_0}} \tag{47}$$

where $M_{p0} = M_p(H_{p0})$, H_{p0} is the solution of [11]. In the case under consideration the propagation speed is proportional to the magnetization of solids. The linear scale of damping is given by

$$\frac{d_p^2 \epsilon_0 \sqrt{2\mu_0 \rho_p^3 \alpha_0} M_{p0}}{18\rho_f \nu (\rho_p \epsilon_0 + \rho_f \alpha_0)\sqrt{3 + (1 + 2\alpha_0)\hat{\chi}_0}} \tag{48}$$

When disturbances propagating with the speed c_1 damp, the main role in a wave propagation process is played by disturbances propagating with the speed c_0 which do not depend on the magnetic properties. The equation of linear wave propagating with the speed c_0 is of the form

$$\eta_t + c_0 \eta_z = (\kappa \mathrm{Fr})^{-1} Q(\alpha_0, \gamma)\eta_{zz} \tag{49}$$

While [45] is valid, the function $Q(\alpha_0, \gamma)$ can be written as follows:

$$Q = \frac{\gamma \alpha_0 \epsilon_0^2}{(\epsilon_0 + \mathrm{De}\alpha_0)^2} - \frac{\alpha_0^2 (n + 2)}{\epsilon_0^2} \tag{50}$$

The RHS of [49] is responsible for diffusive effects due to the magnetic field. More detailed analysis of such effects is given below. The RHS of [49] can be neglected when $\kappa \mathrm{Fr} \gg 1$ (for example, in the case of small Reynolds numbers). In this case the magnetic field and the magnetic properties of the solid phase do not effect the linear concentration wave propagating with the speed c_0 without damping.

LONG CONCENTRATION WAVES

We start with the dimensionless combined momentum equation following from [17] and [18] in the form

$$\begin{aligned} &\mathrm{Fr}^{-1}[(v_p)_t + v_p(v_p)_z - \mathrm{De}((v_f)_t + v_f(v_f)_z) + \alpha^{-1}(p_p)_z)] \\ &= -(1 - \mathrm{De}) + \kappa\epsilon^{-1}\Phi(\epsilon)(v_f - v_p) + mM_p(H_p)(H_p)_z \end{aligned} \tag{51}$$

Now we assume that the characteristic wavelength L is large such that

$$\mathrm{Fr}(1 - \mathrm{De}) = \frac{gL(\rho_p - \rho_f)}{\rho_p U^2} \gg 1 \tag{52}$$

From the momentum equations in the steady state it follows $\kappa = O(1)$.

Since the effective pressure of the solid phase $p_p = O(\rho_p u^2)$ (see, for example, Nigmatulin 1978), the left hand side of [51] can be neglected for long waves (i.e. while $\mathrm{Fr}(1 - \mathrm{De}) \gg 1$) irrespectively to a form of the equation of state of the solid phase.

Below we assume that the magnetic field is strong such that $H_{p0} > H_s$ so that the solid material is magnetically saturated and $M_p = M_s = \text{const}$. Now the field gradient can be related to the concentration gradient in accordance with [12], so that [51] can be written as

$$(1 - v_p)\phi(\epsilon) + \frac{\gamma}{\kappa \mathrm{Fr}}\epsilon_z = K \qquad [53]$$

where

$$K = \frac{\rho_p(\rho_p - \rho_f)d_p^2}{18\rho_f^2 \nu U} \qquad [54]$$

and the parameter γ, defined by [26], for the saturated bed is

$$\gamma = \frac{2\mu_0^2 M_s}{3\rho_p^2 U} \qquad [55]$$

Incorporating [53] into the mass conservation equation for the solid phase [16] gives the following equation of propagation of a non-linear long concentration (voidage) wave:

$$\frac{\partial \epsilon}{\partial t} + c(\epsilon)\frac{\partial \epsilon}{\partial z} - \frac{\gamma}{\kappa \mathrm{Fr}}\frac{\partial}{\partial z}\left(\frac{1-\epsilon}{\phi(\epsilon)}\frac{\partial \epsilon}{\partial z}\right) = 0 \qquad [56]$$

Equation [56] is actually the Burgers equation with the non-linear diffusive term. Here $c(\epsilon)$ is the speed of kinematic concentration wave found by Sergeev (1985) as

$$c = 1 + \mathrm{K}\epsilon^{n+1}[n + 2 - (n+3)\epsilon] \qquad [57]$$

It should be noted that $c(\epsilon)$ does not depend on the magnetic properties. The characteristic speed c as a function of void fraction ϵ is represented in figure 9 at $K = 1$. Here ϵ_* corresponds to the maximum of $c(\epsilon)$. From [57] we immediately find:

$$\epsilon_* = \frac{n+1}{n+3}, \quad \alpha_* = \frac{2}{n+3} \qquad [58]$$

For the Richardson–Zaki correlation ($n = 2.8$) from [58] it follows that $\epsilon_* \simeq 0.655$, $\alpha_* \simeq 0.345$.

Now we consider finite small-amplitude waves. Introducing the concentration (voidage) disturbance η of the uniform state so that the solid concentration and the void fraction are

$$\alpha = \alpha_0 + \eta, \quad \epsilon = \epsilon_0 - \eta \qquad [59]$$

we linearize the last term in [56] taking into account that $\eta_z \ll \eta$ for long waves.

To analyse an effect of magnetic properties and the magnetic field on the propagating disturbance, below we return to the dimensional variables.

After linearization of the diffusive term we obtain Burgers equation

$$\frac{\partial \eta}{\partial t} + Uc(\epsilon_0 - \eta)\frac{\partial \eta}{\partial z} - \frac{\mu_0 M_s^2 d_p^2}{27\rho_f \nu}\frac{1-\epsilon_0}{\phi(\epsilon_0)}\frac{\partial^2 \eta}{\partial z^2} = 0 \qquad [60]$$

Now we linearize the characteristic speed $c(\epsilon)$ such that

$$c(\epsilon_0 - \eta) = c_0 + \beta\eta \qquad [61]$$

where $c_0 = c(\epsilon_0)$ is given by [32] and

$$\beta = -\left(\frac{\mathrm{d}c}{\mathrm{d}\epsilon}\right)_{\epsilon=\epsilon_0} = \frac{(n+2)[(n+3)\epsilon_0 - (n+1)]}{\epsilon_0^2} \qquad [62]$$

Here $\beta > 0$ as $\epsilon_* < \epsilon_0 < 1$ $(0 < \alpha_0 < \alpha_*)$ and $\beta < 0$ as $0 < \epsilon_0 < \epsilon_*$ $(\alpha_* < \alpha_0 < 1$, respectively). We note that for such a linearization the closest vicinity to ϵ_* should be withdrawn from the consideration.

We should underline here that the problem on propagation of non-linear waves in a fluidized bed of magnetic particles reduces to Burgers equation even in the simplest long-wave approximation.

We are reminded that propagation of long concentration waves in a fluidized bed of magnetically neutral particles is described by an equation of the form

$$\frac{\partial \epsilon}{\partial t} + c(\epsilon)\frac{\partial \epsilon}{\partial z} = 0$$

(see, for example, Fanucci *et al.* 1979; Kluwick 1983; Liu 1983; Needham & Merkin 1983; Sergeev 1985).

The pattern of long wave propagation is convenient to be analysed with the help of the Cauchy problem. An initial concentration disturbance is imposed below in the form of an isolated pulse $\eta(z, 0) = \eta_0 S(z)$ with the amplitude η_0 and the characteristic length L (see Figure 10). Far from the initial disturbance the wave front is described by the following expressions (Witham 1974):

for the "right" branch of the characteristic speed (at $\epsilon_0 > \epsilon_*$):

$$\eta = \frac{z}{U\beta t} - \frac{c_0}{\beta} \quad \text{as} \quad 0 < z - Uc_0 t < \sqrt{2\eta_0 \beta U L t}$$

$$\eta = 0 \quad \text{as} \quad z - Uc_0 t > \sqrt{2\eta_0 \beta U L t} \qquad [63]$$

for the "left" branch (at $\epsilon_0 < \epsilon_*$):

$$\eta = \frac{c_0}{|\beta|} - \frac{z}{U|\beta|t} \quad \text{as} \quad \sqrt{2\eta_0|\beta|ULt} < z - c_0 t < 0$$

$$\eta = 0 \quad \text{as} \quad 0 < z - Uc_0 t < \sqrt{2\eta_0|\beta|ULt} \qquad [64]$$

The form of solution for the right and left branches of characteristic speed $c(\epsilon)$ (for $\epsilon > \epsilon_*$ and $\epsilon < \epsilon_*$, respectively) is schematically represented in Figure 10 in terms of solid concentration α.

When analysing propagation of non-linear concentration waves, the formation of domains with a dramatic change of solid concentration and/or solid concentration discontinuities is of key interest, since such domains and discontinuities can be considered as models of bubble (or slug) boundaries formed in a fluidized bed. The coordinate of the wavefront is determined for the right and left branch of characteristic speed, respectively, as

$$Z = Uc_0 t \pm \sqrt{2\eta_0|\beta|Ut} \qquad [65]$$

For the speed of the "shock" front it yields:

$$D = Uc_0 \pm \sqrt{\eta_0|\beta|UL/(2t)} \qquad [66]$$

It should be noted that for magnetically neutral particles [65] and [66] give the position and the velocity of the propagating discontinuities of the solid phase. These discontinuities given in figure 10 by the dotted lines appear in the front and rear parts of disturbance depending whether the voidage ϵ_0 is higher or lower than ϵ_* (i.e. in the parts of disturbance corresponding to the compression or rarefaction of the solid phase; Sergeev 1985). When particles interact due to their magnetization the obtained formulae give the positions and speeds of wave fronts with a sharp (but smooth) change of solid concentration. Like in Sergeev (1985) we find that the obtained solution describes the formation of "shock fronts" in the two following situations (see figure 10):

(a) In the case when $\epsilon > \epsilon_*$ ($\alpha < \alpha_*$)—for propagation of the compression wave of the solid phase.
(b) In the case where $\epsilon < \epsilon_*$ ($\alpha > \alpha_*$)—for propagation of the rarefaction wave of the solid phase.

The amplitude η^* and the thickness δ of the "shock" front follow from solution of [60] with the proper initial and boundary conditions as

$$\eta^* = \sqrt{\frac{2\eta_0|\beta|L}{Ut}} \qquad [67]$$

$$\delta = \frac{\mu_0 M_s^2 d_p^2 (1-\epsilon_0)\sqrt{t}}{27\sqrt{2}\rho_f \nu \phi(\epsilon_0)\sqrt{\eta_0|\beta|UL^3}} \qquad [68]$$

The obtained formulae determine the structure of the "shock" front. In particular, [68] shows that the thickness of the "shock" front is proportional to the square of the magnetization of the solid material, growing as $\sqrt{t}$, does not depend on the solid density but is proportional to the square of the diameter of solid particles.

In conclusion we note that the formation of one-dimensional fronts analysed in the present work can be considered as a model for the formation of bubbles in a fluidized bed, so that the analysed effect of "eroding" of the front and the obtained decrease of the front amplitude with time can be used for a qualitative explanation of the phenomenon of suppression of bubbles in a bubbling fluidized bed.

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